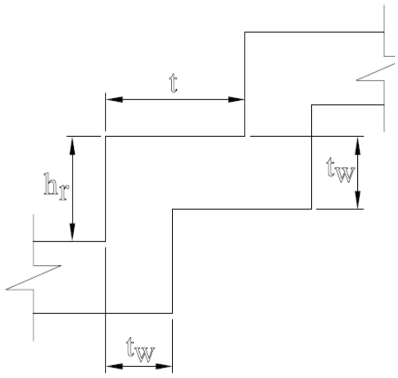
HELICAL STAIRCASE

Reference : "Reinforced Concrete Designer's Handbook" by 'Charles E. Reynolds and James C. Steedman', Tenth Edition, Page No.389

INPUT



Inner Radius, $R_i := 780mm$

Outer Radius, $R_o := 1880mm$

Height between two floor levels, $H_f := 3.205m$

Tread, $t := 250mm$

Rise, $h_r := 170mm$

Width of the staircase, $b := 1100mm$

Thickness of the slab, $t_w := 100mm$

Density of concrete, $\rho := 25 \frac{kN}{m^3}$

DL of stair, $DL_s := \frac{(t \cdot t_w + t_w \cdot h_r) \cdot \rho \cdot b}{t}$ $\quad DL_s = 4.62 \frac{kN}{m}$

Selfweight of finishes (50 bed, stone finish&ceiling), $F_w := 1.65 \frac{kN}{m^2}$

DL of finishes per metre run, $DL_f := \frac{(t + h_r) \cdot b \cdot F_w}{t}$ $\quad DL_f = 3.049 \frac{kN}{m}$

Total DL, $DL := DL_s + DL_f$ $\quad DL = 7.669 \frac{kN}{m}$

Live load for stairs, $LL_s := 2 \frac{kN}{m^2}$

Total LL, $LL := LL_s \cdot b$ $\quad LL = 2.2 \frac{kN}{m}$

Total loading per unit length projected along the centre-line of load, $n := 1.35 \cdot DL + 1.5 \cdot LL$ $\quad n = 13.653 \frac{kN}{m}$

Radius of centre-line of loading, $R_1 := \left(\frac{2}{3}\right) \cdot \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}$

$R_1 = 1.406 m$

Raduis of centre-line of steps, $R_2 := \frac{R_o + R_i}{2}$

$R_2 = 1.33\,m$

Angle subtended in plan between point considered and midpoint of the stair,

$$\theta := \begin{pmatrix} 0 \\ 15 \\ 30 \\ 45 \\ 60 \\ 75 \\ 97.5 \end{pmatrix} deg \qquad \theta = \begin{pmatrix} 0 \\ 0.262 \\ 0.524 \\ 0.785 \\ 1.047 \\ 1.309 \\ 1.702 \end{pmatrix} \quad i := 0..6$$

Total angle subtended by helix in plan,

$\beta := 195deg$

$\beta = 3.403$

Slope of tangent to helix centre-line measured from horizontal,

$fi := \frac{H_f}{3.14 \cdot R_2}$ $\qquad fi = 0.767$

$atan(fi) = 37.504\,deg$

$\phi := atan(fi)$

$\phi = 37.504\,deg$

$\phi = 0.655$

To find the design coefficients k1, k2 & k3

Width of the stair, $b = 1.1\,m$

Thickness of the stair, $h := t_w$

$\left(\frac{b}{h}\right) = 11$ $\qquad \left(\frac{R_1}{R_2}\right) = 1.057$

From Curve

$k_1 := -0.049$ $\quad k_2 := 0.853$ $\quad k_3 := -0.141$

RESULTS

Redundant moment acting tangentially at mid span,

$$M_o := k_1 \cdot n \cdot R_2^{\,2}$$

$$M_o = -1.183\,kN \cdot m$$

Horizontal redundant force at midspan,

$$H := k_2 \cdot n \cdot R_2$$

$$H = 15.49\,kN$$

Vertical moment at support, $M_{vs} := k_3 \cdot n \cdot R_2^{\,2}$ $M_{vs} = -3.405\,kN \cdot m$

For different values of θ_i,

Lateral moment,

$$M_{n_i} := M_o \cdot \sin(\theta_i) \cdot \sin(\phi) - H \cdot R_2 \cdot \theta_i \cdot \tan(\phi)\cos(\theta_i) \cdot \sin(\phi) \; ... \\ + -H \cdot R_2 \cdot \sin(\theta_i) \cdot \cos(\phi) + n \cdot R_1 \cdot \sin(\phi)\left(R_1 \cdot \sin(\theta_i) - R_2 \cdot \theta_i\right)$$

$$M_n = \begin{pmatrix} 0 \\ -6.668 \\ -12.82 \\ -18.002 \\ -21.866 \\ -24.22 \\ -24.94 \end{pmatrix} kN \cdot m$$

Torsional moment,

$$T_i := \left(M_o \cdot \sin(\theta_i) - H \cdot R_2 \cdot \theta_i \cdot \cos(\theta_i) \cdot \tan(\phi) + n \cdot R_1^{\,2} \cdot \sin(\theta_i) - n \cdot R_1 \cdot R_2 \cdot \theta_i\right) \cdot \cos(\phi) \; ... \\ + H \cdot R_2 \cdot \sin(\theta_i) \cdot \sin(\phi)$$

$$T = \begin{pmatrix} 0 \\ 0.07 \\ 0.214 \\ 0.47 \\ 0.813 \\ 1.126 \\ 1.051 \end{pmatrix} kN \cdot m$$

Vertical moment,

$$M_{y_i} := M_o \cdot \cos(\theta_i) + H \cdot R_2 \cdot \tan(\phi) \cdot (\theta_i \cdot \sin(\theta_i)) - n \cdot R_1^{2} \cdot (1 - \cos(\theta_i))$$

$$M_y = \begin{pmatrix} -1.183 \\ -0.991 \\ -0.501 \\ 0.04 \\ 0.255 \\ -0.315 \\ -3.677 \end{pmatrix} kN \cdot m$$

Thrust, $N_{1_i} := -H \cdot \sin(\theta_i) \cdot \cos(\phi) - n \cdot R_1 \cdot \theta_i \cdot \sin(\phi)$

$$N_1 = \begin{pmatrix} 0 \\ -6.24 \\ -12.263 \\ -17.867 \\ -22.879 \\ -27.166 \\ -32.069 \end{pmatrix} kN$$

Lateral shearing force across stair,

$$V_{n_i} := n \cdot R_1 \cdot \theta_i \cdot \cos(\phi) - H \cdot \sin(\theta_i) \cdot \sin(\phi)$$

$$V_n = \begin{pmatrix} 0 \\ 1.546 \\ 3.258 \\ 5.291 \\ 7.779 \\ 10.823 \\ 16.562 \end{pmatrix} kN$$

Radial horizontal shearing force,

$$V_{h_i} := H \cdot \cos(\theta_i)$$

$$V_h = \begin{pmatrix} 15.49 \\ 14.962 \\ 13.414 \\ 10.953 \\ 7.745 \\ 4.009 \\ -2.022 \end{pmatrix} kN$$

Reinforcement due to vertical moment

$d_c := 25mm$ $\quad f_{ck} := 40\frac{N}{mm^2}$ $\quad f_y := 460\frac{N}{mm^2}$

$d := t_w - d_c$ $\quad d = 75\,mm$

Limiting Moment, $M_{ulim} := 0.167 \cdot f_{ck} \cdot b \cdot d^2$ $\quad M_{ulim} = 41.3\,kN \cdot m$

Tension reinforcement, $M_{des} := \left|\left(\min\left(M_y, M_{ulim}\right)\right)\right|$ $\quad M_{des} = 3.7\,kN \cdot m$

$k := \frac{M_{des}}{b \cdot d^2 \cdot f_{ck}}$ $\quad k = 0.015$

Lever arm, $l_a := 0.5 + \sqrt{0.25 - \frac{k}{0.908}}$ $\quad l_a = 0.983$

$l_{amax} := if\left(l_a < 0.95, l_a, 0.95\right)$

$A_{st} := \left(\frac{M_{des}}{l_{amax} \cdot d \cdot 0.87 \cdot f_y}\right)$ $\quad A_{st} = 128.948\,mm^2$

Minimum reinforcement, $f_{ctm} := 3.5\frac{N}{mm^2}$ Table 3.1 (EN 1992-1-1:2004)

$A_{smin} := 0.26 \cdot \frac{f_{ctm}}{f_y} \cdot b \cdot d$ $\quad A_{smin} = 163.207\,mm^2$

$\phi := 8mm$ $\quad Spacing := \frac{1000mm \cdot \frac{\pi \cdot \phi^2}{4}}{A_{st}}$ $\quad Spacing = 389.812\,mm$

Provide $\phi := 8mm$ **dia** $s := 200mm$ $\quad A_{s1} := \frac{1000mm}{s} \cdot \frac{\pi \cdot \phi^2}{4}$ $\quad A_{s1} = 251.327\,mm^2$

Reinforcement due to lateral moment

$M_{des} := \left|\left(\min\left(M_n, M_{ulim}\right)\right)\right|$ $\quad M_{des} = 24.9\,kN \cdot m$

$k := \frac{M_{des}}{1m \cdot d^2 \cdot f_{ck}}$ $\quad k = 0.111$

Lever arm, $l_a := 0.5 + \sqrt{0.25 - \frac{k}{0.908}}$ $\quad l_a = 0.858$

$l_{amax} := if\left(l_a < 0.95, l_a, 0.95\right)$

$A_{st} := \left(\frac{M_{des}}{l_{amax} \cdot d \cdot 0.87 \cdot f_y}\right)$ $\quad A_{st} = 968.8\,mm^2$

Area of steel for one tread $A_{tr} := A_{st} \cdot \frac{(t_w + t)}{1m}$ $A_{tr} = 339.086\ mm^2$

Area of one bar $A_b := \frac{A_{tr}}{4}$ $A_b = 84.772\ mm^2$

Provide $\phi := 12mm$ **dia 4 Nos** $A_{s2} := \frac{\pi \cdot \phi^2}{4}$ $A_{s2} = 113.097\ mm^2$

Shear Reinforcement

Flexural Reinforcement provided, $A_{s1} = 251.327\ mm^2$

$\rho := if\left(\frac{A_{s1}}{b \cdot d} \le 0.02, \frac{A_{s1}}{b \cdot d}, 0.02\right)$ $\rho = 3.046 \times 10^{-3}$

$\upsilon := 0.699 - \frac{f_{ck} \cdot \frac{mm^2}{N}}{200}$ $\upsilon = 0.499$

$\upsilon := if(\upsilon < 0.5, 0.5, \upsilon)$ $\upsilon = 0.5$

$v_{max} := 0.3 \cdot \upsilon \cdot f_{ck}$ $v_{max} = 6 \frac{N}{mm^2}$

$v_c = 0.814 \frac{N}{mm^2}$

$V_{Rd2} := v_{max} \cdot b \cdot d$ $V_{Rd2} = 495\ kN$

$V_n := max(V_n)$ $V_n = 16.562\ kN$

Applied shear stress $\tau_v := \frac{V_n}{b \cdot d}$ $\tau_v = 0.201 \frac{N}{mm^2}$

Shear stress due to torsion,

$T := max(T)$ $T = 1.126\ kN \cdot m$ $\tau_t := \frac{T}{0.23 b^2 t_w}$ $\tau_t = 0.04 \frac{N}{mm^2}$

$v := \tau_v + \tau_t$ $v = 0.241 \frac{N}{mm^2}$

As $v < v_c$ safe for shear.